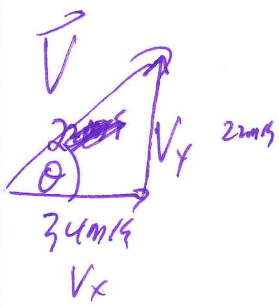
Projectile Motion

Projectiles Launched at an Angle

1) A projectile is fired so that it has an initial vertical velocity of 22 m/s, and an initial horizontal velocity of 34 m/s.

   a. What is the total initial velocity of the projectile? [41 m/s [33°]]



$V = \sqrt{V_x^2 + V_y^2}$

$V = \sqrt{(34 m/s)^2 + (22 m/s)^2}$

$V = 40.4969134 m/s$

$\theta = \arctan\left(\frac{22 m/s}{34 m/s}\right)$

$\theta = 32.905°$

$\vec{V} = 40 m/s$ [33° up from horizontal]

   b. Where it is (x and y) when it has been traveling for 1.3 seconds. [44.2 m, 20.3 m]

Δd_x

$\vec{V}_x = 34 m/s$

$\Delta t = 1.3 s$

$\Delta d_x = ?$

$\Delta \vec{d}_x = \vec{V}_x \Delta t$

$\Delta \vec{d}_x = 34 m/s (1.3 s)$

$\Delta \vec{d}_x \approx 44 m$

Δd_y

$\vec{V}_{1y} = 22 m/s$

$\vec{a}_y = -9.81 m/s$

$\Delta t = 1.3 s$

$\Delta d_y = ?$

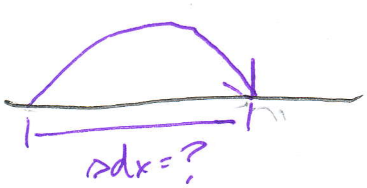
$\Delta \vec{d}_y = \vec{V}_{1y} \Delta t + \frac{1}{2} \vec{a}_y (\Delta t)^2$

$\Delta \vec{d}_y = 22 m/s (1.3 s) + \frac{1}{2}(-9.81 m/s^2)(1.3 s)^2$

$\Delta \vec{d}_y = 20.31055 m$

$\Delta \vec{d}_y \approx 20 m$

   c. What is the total distance it travels in the x direction if it lands at the same height it starts at. [152 m]



$\vec{V}_x = 34 m/s$

$\Delta t_t = ?$

$\vec{V}_{1y} = 22 m/s$

$\Delta \vec{d}_y = 0$

$\vec{a}_y = -9.81 m/s^2$

Find Δt:

$\Delta \vec{d}_y^{\,0} = \vec{V}_{1y} \Delta t + \frac{1}{2} \vec{a}_y (\Delta t)^2$

$\frac{0}{\Delta t} = \frac{\vec{V}_{1y} \Delta t}{\Delta t} + \frac{\frac{1}{2} \vec{a}_y (\Delta t)^2}{\Delta t}$

$0 = \vec{V}_{1y} + \frac{1}{2} \vec{a}_y (\Delta t)$

$\frac{-2 V_{1y}}{a_y} = \Delta t$

$\Delta t_t = \frac{-2(22 m/s)}{-9.81 m/s^2}$

$\Delta t_t = 4.48522 s$

Find Δd_x (Range):

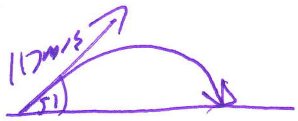
$\Delta \vec{d}_x = \vec{V}_x \Delta t_t$

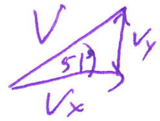
$\Delta \vec{d}_x = 34 m/s (4.48522 s)$

$\Delta d_x \approx 152 m$

2) A trebuchet fires a projectile at an angle of 51° with a speed of 117 m/s. Find

a. The time the projectile is in the air if it is fired on level ground. [18.6 s]





$\Delta t = ?$

$\vec{V}_{y1} = \vec{V}_1 \sin\theta$

$= 117\,m/s(\sin 51°)$

$\vec{V}_{y1} = 90.926\,m/s$

$\vec{a}_y = -9.81\,m/s$

$\vec{V}_{y2} = -90.926\,m/s$

$\vec{a}_y = \frac{\Delta \vec{V}_y}{\Delta t}$

$\Delta t = \frac{\Delta \vec{V}_y}{\vec{a}_y}$

$\Delta t = \frac{-90.926\,m/s - 90.926\,m/s}{-9.81\,m/s^2}$

$\Delta t = 18.537\,s$

$\Delta t \approx 18.6\,s$

b. The horizontal distance it covers. [1370 m]

$\Delta d_x = ?$

$\vec{V}_x = \vec{V}_1 \cos\theta$

$= 117\,m/s(\cos(51°))$

$\vec{V}_x = 73.631\,m/s$

$\Delta t = 18.537\,s$

$\Delta \vec{d}_x = \vec{V}_x \Delta t$

$\Delta \vec{d}_x = 73.631\,m/s(18.537\,s)$

$\Delta \vec{d}_x = 1364.898\,m$

$\Delta \vec{d}_x \approx 1370\,m$

c. The maximum height reached. [422 m]

$\Delta d_y = ?$

$\vec{V}_{y1} = 90.926\,m/s$

$\vec{V}_{y2} = 0\,m/s$

$\vec{a}_y = -9.81\,m/s^2$

$\vec{V}_{y2}^2 = \vec{V}_{y1}^2 + 2\vec{a}_y \Delta d_y$

$-\vec{V}_{y1}^2 = 2\vec{a}_y \Delta d_y$

$\frac{-\vec{V}_{y1}^2}{2\vec{a}_y} = \Delta d_y$

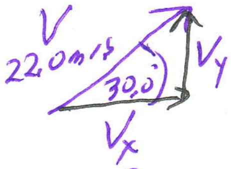
$\Delta \vec{d}_y = \frac{-(90.926\,m/s)^2}{2(-9.81\,m/s)}$

$\Delta \vec{d}_y = 421.3832\,m$

~~$\Delta d_y \approx 420$~~

$\Delta \vec{d}_y \approx 421\,m$

3) Baseball players often practise their swing in a batting cage, in which a pitching machine delivers the ball. If the baseball is launched with an initial velocity of 22.0 m/s [30.0°] and the player hits it at the same height from which it was launched, for how long is the baseball in the air on its way to the batter? [2.24 s]



$\Delta t = ?$

$V_{y1} = \sin 30.0° (22.0 m/s)$

$\vec{V}_{y1} = 11.0 \text{ m/s}$

$\vec{a}_y = -9.81 \text{ m/s}^2$

$\Delta d_y = 0$

$\vec{V}_{y2} = -11.0 \text{ m/s}$

~~$\Delta \vec{d}_y = \vec{V}_{y1} \Delta t + \frac{1}{2}$~~

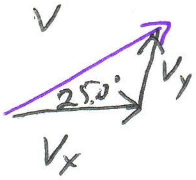
$\vec{a}_y = \frac{\Delta \vec{V}_y}{\Delta t}$

$\Delta t = \frac{\Delta \vec{V}_y}{\vec{a}_y}$

$\Delta t = \frac{-11.0 \text{ m/s} - 11.0 \text{ m/s}}{-9.81 \text{ m/s}^2}$

$\Delta t \approx 2.24 \text{ s}$

4) A paintball directed at a target is shot at an angle of 25.0°. If paint splats on its intended target at the same height from which it was launched, 3.00 s later, find the distance from the shooter to the target. [94.7 m]



$\Delta t = 3.00 \text{ s}$

$\Delta d_x = ?$

$\vec{a}_y = -9.81 \text{ m/s}^2$

$\Delta \vec{d}_y = 0 \text{ m}$

Find V_{y1}:

$\Delta \vec{d}_y^{\,0} = \vec{V}_{y1} \Delta t + \frac{1}{2} \vec{a}_y \Delta t^2$

$\vec{V}_{y1} \Delta t = -\frac{1}{2} \vec{a}_y \Delta t^2$ ($\div \Delta t$)

$\vec{V}_{y1} = -\frac{1}{2} a_y \Delta t$

$\vec{V}_{y1} = -\frac{1}{2}(-9.81 \text{ m/s}^2)(3.00 \text{ s})$

$\vec{V}_{y1} = 14.715 \text{ m/s}$

Find V_x:

$\tan \theta = \frac{V_y}{V_x}$

$V_x = \frac{V_y}{\tan \theta}$

$V_x = \frac{14.715 \text{ m/s}}{\tan 25.0°}$

$V_x = 31.556 \text{ m/s}$

Find Range:

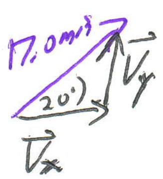
$\Delta \vec{d}_x = V_x \Delta t$

$\Delta \vec{d}_x = 31.556 \text{ m/s} (3.00 \text{ s})$

$\Delta d_x = 94.668 \text{ m}$

$\Delta d_x \approx 94.7 \text{ m}$

5) Determine the height reached by a baseball if it is released with a velocity of 17.0 m/s [20°]. [1.73 m]



$\vec{V}_{y1} = 17.0 \text{ m/s} (\sin 20°)$

$\vec{V}_{y1} = 5.8143 \text{ m/s}$

$\vec{V}_{y2} = 0 \text{ m/s}$

$\vec{a}_y = -9.81 \text{ m/s}^2$

$\Delta \vec{d}_y = ?$

$\vec{V}_{y2}^{2\,0} = \vec{V}_{y1}^2 + 2\vec{a}_y \Delta d_y$

$-\vec{V}_{y1}^2 = 2\vec{a}_y \Delta d_y$

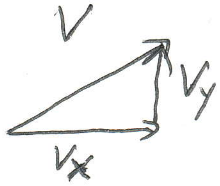
$\frac{-V_{y1}^2}{2\vec{a}_y} = \Delta d_y$

$\Delta \vec{d}_y = \frac{-(5.8143 \text{ m/s})^2}{2(-9.81 \text{ m/s}^2)}$

$\Delta \vec{d}_y = 1.72 \text{ m}$

6) A German U2 rocket from the Second World War had a range of 300 km, reaching a maximum height of 100 km. Determine the rocket's maximum initial velocity. [1.75×10^3 m/s [53.1°]]

Find Δt to Max height:



$V_{y2} =$

$\Delta d_y = 100\,km = 100000\,m$

$\Delta d_x = 300\,km = 300000\,m$

$\vec{a}_y = -9.81\,m/s^2$

$\vec{V}_x = ?$

$\vec{V}_y = ?$

$\vec{V} = ?$

$\Delta t = ?$

7) Platform divers receive lower marks if they enter the water a distance away from the platform, whereas speed swimmers dive as far out into the pool as they can. Compare and contrast the horizontal and vertical components of each type of athlete's motion.

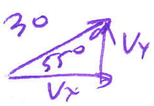
Platform: greater vertical, less horizontal (high launch angle)

Speed: Greater horizontal, less vertical (lower launch angle)

8) For a fixed speed, how does the range depend on the angle, θ?

- greatest at 45°

9) A golf ball is hit with an initial velocity of 30.0 m/s [55°]. What are the ball's range and maximum height? [86.2 m, 30.8 m]



$\vec{V}_x = 30\,m/s(\cos 55°)$

$\vec{V}_x = 17.207\,m/s$

$\vec{V}_{y1} = 30\,m/s(\sin 55°)$

$\vec{V}_{y1} = 24.575\,m/s$

$\vec{a}_y = -9.81\,m/s^2$

$\Delta d_x = ?$

$\Delta d_y = ?$

Time in Flight:

$\vec{V}_{y2} = -24.575\,m/s$

$\Delta t = ?$

$\vec{a}_y = \frac{\Delta \vec{V}_y}{\Delta t}$

$\Delta t = \frac{\Delta \vec{V}_y}{\vec{a}_y}$

$\Delta t = \frac{-24.57\,m/s - 24.57\,m/s}{-9.81\,m/s^2}$

$\Delta t = 5.009\,s$

Range:

$\Delta d_x = \vec{V}_x \Delta t$

$\Delta d_x = 17.207\,m/s(5.009\,s)$

$\Delta d_x \approx 86.2\,m$

Max height:

$\Delta d_y = ?$

$\vec{V}_{y2} = 0\,m/s$

$\Delta t = \frac{5.009\,s}{2} = 2.5045\,s$

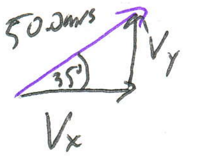
$\Delta \vec{d}_y = V_1 \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$

$\Delta d_y = 24.575(2.5045) + \frac{1}{2}(-9.81)(2.5045)^2$

$\Delta d_y \approx 30.8\,m$

10) During the Apollo 14 mission, Alan Shepard was the first person to hit a golf ball on the Moon. If a golf ball was launched from the Moon's surface with a velocity of 50.0 m/s [35°] and the acceleration due to gravity on the Moon is -1.61 m/s^2,

a. how long was the golf ball in the air? [35.6 s]



$\vec{V}_{y1} = \vec{V}\sin\theta$

$\vec{V}_{y1} = 50.0\,m/s(\sin 35°)$

$\vec{V}_{y1} = 28.6788\,m/s$

$\vec{a}_y = -1.61\,m/s^2$

$\vec{V}_{y2} = -28.6788\,m/s$

$\Delta t = ?$

$\vec{a}_y = \frac{\Delta\vec{V}}{\Delta t}$

$\Delta t = \frac{\Delta\vec{V}}{\vec{a}_y}$

$\Delta t = \frac{-28.678\,m/s - 28.678\,m/s}{-1.61\,m/s^2}$

$\Delta t = 35.6248\,s$

$\Delta t \approx 35.6\,s$

b. what was the golf ball's range? [1.5 km]

$\Delta d_x = ?$

$\vec{V}_x = \vec{V}\cos\theta$

$\vec{V}_x = 50.0\,m/s[\cos(35°)]$

$\vec{V}_x = 40.9576\,m/s$

$\Delta t = 35.6248\,s$

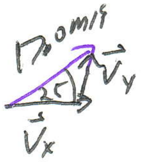
$\Delta\vec{d}_x = \vec{V}_x \Delta t$

$\Delta\vec{d}_x = 40.9576\,m/s(35.6248\,s)$

$\Delta\vec{d}_x \approx 1459\,m$

$\Delta d_x \approx 1500\,m \approx 1.5\,Km$

11) A football is thrown to a moving receiver. The football leaves the quarterback's hands 1.75 m above the ground with a velocity of 17.0 m/s [25°]. If the receiver starts 12.0 m away from the quarterback along the line of flight of the ball when it is thrown, what constant velocity must she have to get to the ball at the instant it is 1.75 m above the ground? [7.21 m/s]



$\vec{V}_{y1} = 17\sin(25°)$

$V_{y1} = 7.1845\,m/s$

$\vec{a}_y = -9.81\,m/s^2$

$\Delta t = ?$

$\vec{V}_{y2} = -7.1845\,m/s$

$\vec{V}_x = 17\cos(25°)$

$\vec{V}_x = 15.4072\,m/s$

Find time of flight:

$\Delta t = \frac{\Delta\vec{V}}{\vec{a}_y}$

$\Delta t = \frac{-7.1845\,m/s^2 - 7.1845\,m/s^2}{-9.81\,m/s^2}$

$\Delta t = 1.4647\,s$

Find Range:

$\Delta\vec{d}_x = \vec{V}_x \Delta t$

$\Delta\vec{d}_x = 15.4072\,m/s(1.4647\,m/s)$

$\Delta\vec{d}_x = 22.5669\,m$

Find distance receiver needs to cover:

$22.5669\,m/s - 12.0\,m$

$\Delta d_x \Rightarrow 10.5669\,m$

Find speed needed to cover distance:

$\vec{V}_{xR} = \frac{\Delta\vec{d}_x}{\Delta t}$

$\vec{V}_{xR} = \frac{10.5669\,m}{1.4647\,s}$

$\vec{V}_{xR} \approx 7.21\,m/s$